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OUTPUT TRACKING BY STATE FEEDBACK FOR HIGH-ORDER NONLINEAR SYSTEMS WITH TIME-DELAY

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ABSTRACT

This paper addresses the problem of global practical output tracking for a class of high-order time-delay uncertain non-linear systems via state feedback. On the basis of the homogeneous domination technique, under mild conditions on the system nonlinearities involving time delay, we construct a homogeneous state feedback controller with an adjustable scaling gain. With the aid of a homogeneous Lyapunov-Krasovskii functional, the scaling gain is adjusted to dominate the time-delay nonlinearities bounded by homogeneous growth conditions and make the tracking error arbitrarily small while all the states of the closed-loop system remain to be bounded. Finally, a simulation example is given to illustrate the effectiveness of the tracking controller.

Keywords: *Output Tracking, Time-Delay Nonlinear Systems, State Feedback, Homogeneous Domination Technique*

1. INTRODUCTION

This paper addresses the global practical output tracking problem for a class of uncertain nonlinear systems with time-delay which is described by

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t)^{p_i} + \varphi_i(t, x(t), x(t-d), u(t)), \\ &\qquad i = 1, \dots, n-1, \\ \dot{x}_n &= u + \varphi_n(t, x(t), x(t-d), u(t)), \\ y &= x_1(t), \end{aligned} \tag{1}$$

where $x(t) := (x_1(t), \dots, x_n(t))^{\mathrm{T}} \in R^n$, $u(t) \in R$, and $y(t) \in R$ are the system state, control input and output, respectively. The constant $d \geq 0$ is a given time-delay of the system, for $i = 1, \dots, n$, and the system initial condition is $x(\theta)=\varphi_0(\theta),\ \ \theta\in[-d,0]$. The terms $\varphi_i(\cdot)$ represent nonlinear perturbations that are continuous functions and $p_i\in R_{odd}^{\geq 1}:=\{p/q\in[0,\infty): p$ and q are odd integers, $p\geq q$ $\}$ $(i=1,\ldots,n-1)$ are said to be the high orders of the system.

Global practical output tracking problem of nonlinear systems is one of the most important and challenging problems in the field of nonlinear control and has received a great deal of attention. By posed some conditions on system growth and power order, the practical output tracking problem of system (1) has been well-studied and a number of interesting results have been achieved over the past years, see [1-10], as well as the references therein. However, the aforementioned results have not considered the time-delay effect which is actually very common in state, input, and output due to the time consumed in sensing, information transmitting, and controller computing. It is well known that time-delay phenomena exist in many practical systems such as electrical networks, microwave oscillator, and hydraulic systems, etc., due to the presence of time delay in systems, it often significant effect on system performance, it often causes deterioration of control system performance and may induce instability, oscillation. Therefore, the study of output tracking and stabilization of time-delay nonlinear systems has important practical significance and has received much attention in recent years. In recent years, by employing the Lyapunov-Krasovskii method to deal with the time-delay, control theory, and techniques for stabilization problem of time-delay nonlinear systems were greatly developed and advanced methods have been made; see, for instance, [11-20] and reference therein. Compared with study the stabilization problem contain time-delay, the theory of output tracking control developed slower. In the case when the nonlinearities contain time-delay, for the output tracking problems, some interesting results have been obtained [21-24]. However, the contributions only considered the case $p_i=1$ for the system (1). When the system under consideration is inherently time-delay non-linear, i.e. the case $p_i>1$, the problem becomes more complicated and difficult to solve, and not many results have been reported in the literature for such a nonlinear system. To the best of our knowlege, due to no unified method being applicable to nonlinear control design, many interesting and important output tracking control problems for time delay inherently nonlinear systems unsolved yet.

In this paper, we aim to solve the problem by using the state feedback domination approach. First, on the basis of the homogeneous domination technique [26, 27, 14], under mild conditions on the system nonlinearities involving time delay, we construct a homogeneous state feedback controller with an adjustable scaling gain. Then, with the aid of a homogeneous Lyapunov-Krasovskii functional, the scaling gain is adjusted to dominate the time-delay nonlinearities bounded by homogeneous growth conditions and make the tracking error arbitrarily small while all the states of the closed-loop system remain to be bounded. The simulation results show the effectiveness of the proposed method. The main contribution of this paper is highlighted as follows. By comparison with the existing results in [21-24], due to the appearance of high-order, time-delay and nonlinear assumption, how to construct an appropriate Lyapunov-Krasovskii functional for system (1) is a nontrivial work.

2. MATHEMATICAL PRELIMINARIES

At first, we give the following notations which will be used in this study.

Notations: R^n denotes the real n-dimensional space and $R^+ := [0,\infty)$. For any vector $x := (x_1,\ldots,x_n)^T \in R^n$, denote $\bar{x}_i := (x_1,\ldots,x_i)^T \in R^i$, $i=1,\ldots,n$ and $\|x\|$ denotes Euclidean norm of x . A function $f : R^n \to R$ is said to be C^k-function, if its partial derivatives exist and are continuous up to order k, $1 \le k < \infty$. A C^0 function means it is continuous. A C^∞ function means it is *smooth*, that is, it has continuous partial derivatives of any order. Besides, the arguments of functions (or functionals) are sometimes omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by $f(x)$, $f(\cdot)$, or f .

Now, we collect the definition of homogeneous function and several useful lemmas.

Definition1[25]. For a set of coordinates $x = (x_1,\ldots,x_n) \in R^n$ and an n-tuple $r = (r_1,\ldots,r_n)$ of positive real numbers we introduce the following definitions.

(i) *A dilation* $\Delta_s(x)$ is a mapping defined by

$$\Delta_s^r(x) = \left(s^{r_1}x_1,\cdots,s^{r_n}x_n\right), \quad \forall x = (x_1,\ldots,x_n) \in R^n,$$

$\forall s > 0$, where r_i are called *the weights of the coordinate*. For simplicity of notation, the dilation weight is denoted by $\Delta = (r_1,\ldots,r_n)$.

(ii) A function $V \in C(R^n, R)$ is said to be *homogeneous of degree* τ if there is a real number $\tau \in R$ such that

$$V(\Delta_s^r(x)) = s^\tau V(x_1,\cdots,x_n), \quad \forall x \in R^n - \{0\}.$$

(iii) A vector field $f \in C(R^n, R^n)$ is said to be *homogeneous of degree* τ if the component f_i is *homogeneous of degree* $\tau + r_i$ for each *i*, that is,

$$f_i(\Delta_s^r(x)) = s^{\tau+r_i} f_i(x_1,\cdots,x_n), \quad \forall x \in R^n,$$

$\forall s > 0$, for $i = 1,\ldots,n$.

(iv) *A homogeneous p -norm* is defined as

$$\|x\|_{\Delta,p} = \left(\sum_{i=1}^n |x_i|^{p/r_i}\right)^{1/p}, \ \forall x \in \mathrm{R}^n, \ p \ge 1.$$

For the simplicity, write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Next, we introduce several technical lemmas which will play an important role and be frequently used in the later control design.

Lemma1[25]. Denote $\Delta = (r_1,\ldots,r_n)$ as dilation weight, and suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions with degree τ_1 and τ_2, respectively. Then, $V_1(x)V_2(x)$ is also homogeneous function with degree of $\tau_1+\tau_2$ with respect to the same dilation Δ.

Lemma2[25]. Suppose $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then, the following (i) and (ii) hold:

(i) $\partial V / \partial x_i$ is also homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i .

(ii) There is a constant $\sigma > 0$ such that

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$V(x) \le \sigma \|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive definite, there is a constant $\rho > 0$ such that $\rho \|x\|_\Delta^\tau \le V(x)$.

Lemma3[27]. For all $x, y \in R$ and a constant $p \ge 1$ the following inequalities hold:

(i) $|x+y|^p \le 2^{p-1} |x^p + y^p|$,

$$\left(|x|+|y|\right)^{1/p} \le |x|^{1/p} + |y|^{1/p} \le 2^{(p-1)/p} \left(|x|+|y|\right)^{1/p}$$

If $p \in R_{odd}^{\ge 1}$, then

(ii) $|x-y|^p \le 2^{p-1} |x^p - y^p|$ and $|x^{1/p} - y^{1/p}| \le 2^{(p-1)/p} |x-y|^{1/p}$.

Lemma4[25]. Let c, d be positive constants. Then, for any real-valued function $\gamma(x,y) > 0$, the following inequality holds:

$$|x|^c |y|^d \le \frac{c}{c+d} \gamma(x,y) |x|^{c+d} + \frac{d}{c+d} \gamma^{-\frac{c}{d}}(x,y) |y|^{c+d}.$$

This paper deals with the practical output tracking problem by state feedback for time-delay high-order nonlinear systems (1). Here, we give a precise definition of the problem.

The problem of global practical tracking by a state feedback: Consider system (1) and assume that the reference signal $y_r(t)$ is a time-varying C^1-bounded function on $[0,\infty)$. For any given $\varepsilon > 0$, design a state feedback controller having the following structure

$$u(t) = g(x(t), y_r(t)), \tag{2}$$

such that

(i) All the state of the closed-loop system (1) with state controller (2) is well-defined and globally bounded on R^+.

(ii) For any initial condition, there is a finite time $T > 0$, such that

$$|y(t) - y_r(t)| < \varepsilon, \quad \forall t \ge T > 0 \tag{3}$$

In order to solve the global practical output tracking problem, we made the following two assumptions:

Assumption1. There are constants C_1, C_2 and $\tau \ge 0$ such that

$$\begin{aligned} &|\varphi_i(t, x(t), x(t-d), u(t))| \le \\ &C_1 \Big(|x_1(t)|^{(r_i+\tau)/r_1} + |x_2(t)|^{(r_i+\tau)/r_2} + \cdots + |x_i(t)|^{(r_i+\tau)/r_i} \\ &+ |x_1(t-d)|^{(r_i+\tau)/r_1} + \cdots + |x_i(t-d)|^{(r_i+\tau)/r_i} \Big) + C_2 \end{aligned} \tag{4}$$

where

$$r_1 = 1, \quad r_{i+1} p_i = r_i + \tau > 0, \quad i = 1, \ldots, n \tag{5}$$

and $p_n = 1$.

Remark1. In the Assumption1, when $\tau > 0$ is a high-order growth condition which is actually homogeneous (see Definition1) with the dilation (5) (For simplicity, in this paper we assume $\tau = p/q$ with an even integer p and an odd integer q. Therefore, r_i is a ratio of two odd integers). Moreover, when $d = 0$, it reduces assumptions in [4]-[7] and this played an essential role to solve the practical tracking problem by a state or output feedback. Specifically, when $p_i = 1$, $i = 1, \ldots, n$, it encompasses the assumptions in existing results [21].

Assumption2. The reference signal $y_r(t)$ is continuously differentiable. Moreover, there is a known constant $D > 0$, such that

$$|y_r(t)| + |\dot{y}_r(t)| \le D, \quad \forall t \in [0, \infty).$$

3. STATE FEEDBACK TRACKING CONTROL DESIGN

This paper deals with the practical output tracking problem by delay-independent state feedback for high-order time-delay nonlinear systems (1) under Assumptions 1-2. To this end, we first introduce the following coordinate transformation:

Define

$$z_1 := x_1 - y_r, z_i := \frac{x_i}{L^{\kappa_i}}, i = 2,\dots,n, \quad v := \frac{u}{L^{\kappa_n+1}} \quad (6)$$

where $\kappa_1 = 0$, $\kappa_i = (\kappa_{i-1}+1)/p_{i-1}$, $i = 2,\dots,n$ and $L \ge 1$ is a constant (scaling gain) to be determined later. Then, the system (1) can be described in the new coordinates z_i as

$$\begin{aligned} \dot{z}_i &= Lz_{i+1}^{p_i} + \psi_i(t, z(t), z(t-d), v), \\ &\qquad i = 1,\dots,n-1, \\ \dot{z}_n &= Lv + \psi_n(t, z(t), z(t-d), v), \\ y &= z_1 + y_r \end{aligned} \quad (7)$$

where

$$\psi_1(t, z(t), z(t-d), v) = \varphi_1(t, z(t), z(t-d), v) - \dot{y}_r,$$
$$\psi_i(t, z(t), z(t-d), v) = \varphi_i(t, z(t), z(t-d), v)/L^{\kappa_i}, \quad i = 2,\dots,n.$$

Remark 2. We need to emphasize that the gain L creates an extra freedom in control design. As a matter of fact, in the proof of main results, complex uncertainties will inevitably be produced in the amplification of nonlinearities. Hence, the gain L can be used to effectively dominate all the possible uncertainties.

Now, using Assumption 1, Lemma 3 and the fact that $L \ge 1$, the following inequalities can be obtained:

$$|\psi_1(t, z(t), z(t-d), v)| \le C_1\left(|z_1(t)+y_r|^{\frac{r_1+\tau}{r_1}} + |z_1(t-d)+y_r|^{\frac{r_1+\tau}{r_1}}\right) + C_2 + |\dot{y}_r|$$

$$|\psi_i(t, z(t), z(t-d), v)| = \left|\frac{\varphi_i(t, x(t), x(t-d), u)}{L^{\kappa_i}}\right| \le \frac{C_1}{L^{\kappa_i}}\left(\left[|z_1(t)+y_r|^{\frac{r_1+\tau}{r_1}} + \cdots + |L^{\kappa_i} z_i(t)|^{\frac{r_1+\tau}{r_1}}\right] + \left[|z_1(t-d)+y_r|^{\frac{r_1+\tau}{r_1}} + \cdots + |L^{\kappa_i} z_i(t-d)|^{\frac{r_1+\tau}{r_1}}\right]\right) + \frac{C_2}{L^{\kappa_i}}$$

Further, the boundedness of y_r and $\dot{y}_r$ guaranteed by Assumption 2, ensures the existence of constants $\bar{C}_i$, $i = 1, 2$ only depending on constants C_1, C_2, τ, κ_i and L, under which (4) becomes

$$|\psi_1(t, z(t), z(t-d), v)| \le \bar{C}_1\left(|z_1(t)|^{\frac{r_1+\tau}{r_1}} + |z_1(t-d)|^{\frac{r_1+\tau}{r_1}}\right) + \bar{C}_2$$

$$|\psi_i(t, z(t), z(t-d), v)| \le \bar{C}_1 L^{1-\nu_i}\sum_{j=1}^{i}\left(|z_j(t)|^{(r_i+\tau)/r_j} + |z_j(t-d)|^{(r_i+\tau)/r_j}\right) + \frac{\bar{C}_2}{L^{\kappa_i}}, \quad i = 2,\dots,n \quad (8)$$

where $\bar{C}_1 > 0$, $\bar{C}_2 > 0$ and

$$\nu_i := \min\left\{1 - \frac{\kappa_j(r_i+\tau)}{r_j} + \kappa_i, 2 \le j \le i, 1 \le i \le n\right\} > 0$$

are some constants.

Since it can be seen that by definition $r_j := \tau\kappa_j + 1/(p_1 \dots p_{j-1})$ so

$$\kappa_j \frac{r_{i+1} p_i}{r_j} - \kappa_i = \frac{\kappa_j \left(\tau\kappa_i + 1/p_1 \dots p_{i-1} + \tau\right)}{\tau\kappa_j + 1/p_1 \dots p_{j-1}} - \kappa_i$$
$$= \frac{\tau\kappa_j + \kappa_j/p_1 \dots p_{i-1} - \kappa_i/p_1 \dots p_{j-1}}{\tau\kappa_j + 1/p_1 \dots p_{j-1}}$$
$$\leq \frac{\tau\kappa_j}{\tau\kappa_j + 1/p_1 \dots p_{j-1}} < 1,$$
$$j = 2,\dots,i, \quad i = 1,\dots,n.$$

In what follows, we will employ the homogeneous domination approach to construct a global state feedback controller for system (7).

3.1. Stability analysis for Nominal Nonlinear System.

First, we construct a homogeneous state feedback stabilizer (controller) for the nominal nonlinear system without considering the non-linearities of $\psi_i(\cdot)$, $i = 1,\dots,n-1$ in (7), i.e.,

$$\begin{aligned} \dot{z}_i &= L z_{i+1}^{p_i}, \quad i = 1,\dots,n-1, \\ \dot{z}_n &= Lv, \\ y &= z_1 + y_r \end{aligned} \tag{9}$$

Using similar the approach in [14, 26], we can design a homogeneous state feedback stabilizer for (9), which can be described in the following Theorem1.

Theorem1. For a real given number $\tau \geq 0$, there is a homogeneous state feedback controller of degree τ such that the nonlinear systems (9) is globally asymptotically stable.

Proof. To prove the result, we use an inductive argument (recursive design method) to explicitly construct a homogeneous stabilizer for system (9).

Initial step1. Let $\xi_1 = z_1^{\sigma/r_1} - z_1^{*\sigma/r_1}$, where $z_1^* = 0$ and $\sigma \geq \max_{1\leq i\leq n}\{1, \tau + r_i\}$ is a positive number. Choose the Lyapunov function

$$V_1 = W_1 = \int_{z_1^*}^{z_1} \left(s^{\sigma/r_1} - z_1^{*\sigma/r_1}\right)^{(2\sigma-\tau-r_1)/\sigma} ds \text{, i.e.}$$

$$V_1 = W_1 = \int_0^{z_1} s^{(2\sigma-\tau-r_1)/r_1} ds$$

From (9), it follows that

$$\dot{V}_1 \leq -nL\xi_1^2 + L\xi_1^{(2\sigma-\tau-r_1)/\sigma}\left(z_2^{p_1} - z_2^{*p_1}\right) \tag{10}$$

where z_2^* the virtual controller and it is chosen as

$$z_2^* = -n^{\frac{1}{p_1}} z_1^{\frac{r_1+\tau}{p_1}} := -\beta_1^{\frac{r_2}{\sigma}} \xi_1^{\frac{r_2}{\sigma}}, \ \beta_1 = n^{\frac{\sigma}{r_2 p_1}}. \tag{11}$$

Step $k\ (k = 2,\cdots,n)$. Suppose at the *step k-1*, there is a C^1, positive definite and proper Lyapunov function V_{k-1}, and a set of virtual controllers $z_1^*,\dots,z_k^*$ defined by

$$\begin{aligned} &z_1^* = 0, && \xi_1 = z_1^{\sigma/r_1} - z_1^{*\sigma/r_1} \\ &z_2^* = -\beta_1^{r_2/\sigma}\xi_1^{r_2/\sigma}, && \xi_2 = z_2^{\sigma/r_2} - z_2^{*\sigma/r_2} \\ &\vdots && \vdots \\ &z_k^* = -\beta_{k-1}^{r_k/\sigma}\xi_{k-1}^{r_k/\sigma}, && \xi_k = z_k^{\sigma/r_k} - z_k^{*\sigma/r_k} \end{aligned} \tag{12}$$

with $\beta_i > 0,\ 1 \leq i \leq k-1$ being constants, such that

$$\dot{V}_{k-1} \leq -(n-k+2)L\sum_{l=1}^{k-1}\xi_l^2 + L\xi_{k-1}^{(2\sigma-\tau-r_{k-1})/\sigma}\left(z_k^{p_{k-1}} - z_k^{*p_{k-1}}\right) \tag{13}$$

We claim that (13) also holds at *Step k,* i.e., there is a C^1, proper, positive definite Lyapunov function defined by

$$\begin{aligned} V_k(\bar{z}_k) &= V_{k-1}(\bar{z}_{k-1}) + W_k(\bar{z}_k), \\ W_k(\bar{z}_k) &= \int_{z_k^*}^{z_k}\left(s^{\sigma/r_k} - z_k^{*\sigma/r_k}\right)^{(2\sigma-\tau-r_k)/\sigma} ds \end{aligned} \tag{14}$$

and virtual controller $z_{k+1}^* = -\beta_k^{r_{k+1}/\sigma}\xi_k^{r_{k+1}/\sigma}$ such that

$$\dot{V}_k \le -(n-k+1)L\sum_{j=1}^{k}\xi_j^2 + L\xi_k^{(2\sigma-\tau-r_k)/\sigma}\left(z_{k+1}^{p_k}-z_{k+1}^{*p_k}\right). \qquad (15)$$

Since the prove of the claim (15) is very similar [4-5, 14], so omitted here.

Using the inductive argument above, we can conclude that at the n-th step, there exists a state feedback controller of the form

$$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\bar{\beta}_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma} \qquad (16)$$

with the C^1, proper and positive definite Lyapunov function,

$$V_n = \sum_{i=1}^{n}\int_{z_i^*}^{z_i}\left(s^{\sigma/r_i}-z_i^{*\sigma/r_i}\right)^{(2\sigma-\tau-r_i)/\sigma}ds \qquad (17)$$

we arrive at

$$\dot{V}_n \le -L\sum_{j=1}^{n}\xi_j^2, \qquad (18)$$

where $\xi_i = z_i^{\sigma/r_i} - z_i^{*\sigma/r_i}$ and $\bar{\beta}_i = \beta_n \cdots \beta_i,\ i = 1,\dots,n$ are positive constants. Thus, the closed-loop system (9) and (16) is globally asymptotically stable.

3.2. Tracking control design for the time-delay nonlinear system (1)

Now, we are ready to use the homogeneous domination approach to design a global tracking controller for the system (1), i.e., state the following main result in this paper.

Theorem 2. For the time-delay nonlinear system (1) under Assumptions 1-2, the global practical output tracking problem is solvable by the state feedback controller $u = L^{\kappa_n+1}v$ in (7) and (16).

Proof.

Step1. We first prove that u preserves the equilibrium at the origin. From (16), we have

$$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\bar{\beta}_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma} \qquad (19)$$

By which and the definitions of r_i 's and σ, we easily see that $u = L^{\kappa_n+1}v$ is a continuous function of z and $u = 0$ for $z = 0$. This together with Assumption1 implies that the solutions of z system is defined on a time interval $[0, t_M]$, where $t_M > 0$ may be a finite constant or $+\infty$, and u preserves the equilibrium at the origin.

Step 2.

Define the compact notations

$$z = (z_1,\dots,z_n)^T,\quad E(z) = \left(z_2^{p_1},\dots,z_n^{p_{n-1}},v\right)^T \quad \text{and}$$

$$F(z) = \left(\varphi_1, \varphi_2/L^{\kappa_2},\dots,\varphi_n/L^{\kappa_n}\right)^T. \qquad (20)$$

Using the same notation (6) and (20), the closed-loop system (7) - (16) can be written as the following compact form:

$$\dot{z} = LE(z) + F(z) \qquad (21)$$

Moreover, by introducing the dilation weight $\Delta = (r_1,\dots,r_n)$, from Definition 1, it can be shown that V_n is homogeneous of degree $2\sigma - \tau$ with respect to Δ.

Hence, adopting the same Lyapunov function (17) and by Lemm2 and Lemma 3, it can be concluded that

$$\dot{V}_n(z) = L\frac{\partial V_n}{\partial z}E(z) + \frac{\partial V_n}{\partial z}F(z) \le -m_1 L\|z\|_\Delta^{2\sigma} + \sum_{i=1}^{n}\frac{\partial V_n}{\partial z_i}\psi_i \qquad (22)$$

where $m_1 > 0$ is constant.

By (8), Assumption 1 and $L > 1$, we can find constants $\delta_i > 0$ and $0 < \gamma_i \le 1$ such that

$$|\psi_i| \le \bar{C}_1 \sum_{j=1}^{i} L^{\frac{\kappa_j (r_i+\tau)}{r_j - k_i}} \left(|z_j(t)|^{\frac{(r_i+\tau)}{r_j}} + |z_j(t-d)|^{\frac{(r_i+\tau)}{r_j}} \right) + \frac{\bar{C}_2}{L^{\kappa_i}} \le \delta_i L^{1-\gamma_i} \left(\|z(t)\|_\Delta^{r_i+\tau} + \|z(t-d)\|_\Delta^{r_i+\tau} \right) + \frac{\bar{C}_2}{L^{\kappa_i}} \quad (23)$$

and noting that for $i = 1,\ldots,n$, by Lemma2, $\partial V_n / \partial z_i$ is homogeneous of degree $2\sigma - \tau - r_i$,

$$\left| \frac{\partial V_n}{\partial z_i} \right| \le m_2 \|z\|_\Delta^{2\sigma-\tau-r_i}, \quad m_2 > 0 \quad (24)$$

and by

$$m_2 \|z\|_\Delta^{2\sigma-\tau-r_i} \frac{\bar{C}_2}{L^{\kappa_i}} = L^{1-\gamma_i} \|z\|_\Delta^{2\sigma-\tau-r_i} \frac{\omega}{L^{1-\gamma_i+\kappa_i}},$$

$$\omega =: m_2 \bar{C}_2 \le m_2 L^{1-\gamma_i} \frac{2\sigma - \tau - r_i}{2\sigma} \|z\|_\Delta^{2\sigma} + L^{1-\gamma_i} \frac{\tau + r_i}{2\sigma} \left(\frac{\omega}{L^{1-\gamma_i+\kappa_i}} \right)^{2\sigma/(\tau+r_i)}$$

$$\le m_2 L^{1-\gamma_i} \|z\|_\Delta^{2\sigma} + L^{1-\gamma_i} \left(\frac{\omega}{L^{1-\gamma_i+\kappa_i}} \right)^{2\sigma/(\tau+r_i)}$$

$$= m_2 L^{1-\gamma_i} \|z\|_\Delta^{2\sigma} + \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{2\sigma(1-\gamma_i+\kappa_i)/(\tau+r_i)-(1-\gamma_i)}}$$

So,

$$\left| \frac{\partial V_n}{\partial z_i} \psi_i \right| \le m_2 \|z\|_\Delta^{2\sigma-\tau-r_i} \left[\delta_i L^{1-\gamma_i} \left(\|z(t)\|_\Delta^{r_i+\tau} + \|z(t-d)\|_\Delta^{r_i+\tau} \right) + \frac{\bar{C}_2}{L^{\kappa_i}} \right]$$
$$\le m_2 L^{1-\gamma_i} (1+\delta_i) \|z\|_\Delta^{2\sigma} + m_2 L^{1-\gamma_i} (1+\delta_i) \|z\|_\Delta^{2\sigma-\tau-r_i} \|z(t-d)\|_\Delta^{r_i+\tau} + \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{2\sigma(1-\gamma_i+\kappa_i)/(\tau+r_i)-(1-\gamma_i)}} \quad (25)$$
$$\le m_2 (1+\delta_i) L^{1-\gamma_i} \|z\|_\Delta^{2\sigma} + m_2 L^{1-\gamma_i} (1+\delta_i) \|z\|_\Delta^{2\sigma-\tau-r_i} \|z(t-d)\|_\Delta^{r_i+\tau} + \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}},$$

where $\omega =: m_2 \bar{C}_2$, $\frac{2\sigma - \tau - r_i}{2\sigma} \le 1$, $\frac{\tau + r_i}{2\sigma} \le 1$, and $\frac{2\sigma - (1-\gamma_i)}{\tau + r_i} - (1-\gamma_i) \ge 1 + \kappa_i$.

Substituting (25) into (22) yields

$$\dot{V}_n(z) \le -L(m_1 \|z\|_\Delta^{2\sigma} - (1 + m_2(1+\delta) \sum_{i=1}^{n} L^{-\gamma_i} \|z\|_\Delta^{2\sigma} - m_2(1+\delta) \sum_{i=1}^{n} L^{-\gamma_i} \|z\|_\Delta^{2\sigma-r_i-\tau} \|z(t-d)\|_\Delta^{r_i+\tau}) + \sum_{i=1}^{n} \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}} \quad (26)$$

where $\delta = \max\{\delta_i\}$, $i = 1,\ldots,n$.

By Lemma4, there exists a constant $m_3 > 0$ such that

$$m_2(1+\delta) \|z\|_\Delta^{2\sigma-r_i-\tau} \|z(t-d)\|_\Delta^{r_i+\tau} \le \|z\|_\Delta^{2\sigma} + m_3 \|z(t-d)\|_\Delta^{2\sigma}, \quad (27)$$

which yields

$$\dot{V}_n(z) \le -L(m_1 \|z\|_\Delta^{2\sigma} - (2 + m_2(1+\delta)) \sum_{i=1}^{n} L^{-\gamma_i} \|z\|_\Delta^{2\sigma} - m_3 \sum_{i=1}^{n} L^{-\gamma_i} \|z(t-d)\|_\Delta^{2\sigma}) + \sum_{i=1}^{n} \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}} \quad (28)$$

Construct a Lyapunov-Krasovskii functional as follows:

$$V(z(t)) = V_n(z(t)) + U(z(t)),$$
$$V_n = \sum_{i=1}^{n} \int_{z_i^*}^{z_i} \left(s^{\sigma/r_i} - z_i^{*\sigma/r_i} \right)^{(2\sigma-\tau-r_i)/\sigma} ds, \quad (29)$$
$$U(z(t)) = \int_{t-d}^{t} \|z(s)\|_\Delta^{2\sigma} M ds,$$

where M is a positive constant. Let $M = m_3 \sum_{i=1}^{n} L^{1-\gamma_i}$. It follows from (28) and (29) that

$$\dot{V} \le -L\left(m_1 - (2 + m_2(1+\delta) + m_3) \sum_{i=1}^{n} L^{-\gamma_i} \right) \|z(t)\|_\Delta^{2\sigma} + \frac{\rho_1}{L^{1+\gamma}}. \quad (30)$$

Hence, by choosing a large enough L as $L > \max\left\{1, (((2 + m_2(1+\delta) + m_3)/m_1))^{-\gamma}\right\}$, where

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$\gamma=\min_{1\le i\le n}\{\gamma_i\}$ and $\rho_1=\sum_{i=1}^{n}\alpha^{2\sigma/(\tau+r_i)}$ the inequality (30) becomes

$$\dot V(z(t))\le -L\|z(t)\|_\Delta^{2\sigma}+\frac{\rho_1}{L^{1+\gamma}}. \tag{31}$$

In (29), $V_n(z)$ and $U(z)$ are homogeneous of degree $2\sigma-\tau$ and 2σ with respect to Δ, respectively. Therefore, it follows from Lemma2 that there exist positive constants $\lambda_1,\ \lambda_2,\ \varpi_1$ and ϖ_2 such that

$$\lambda_1\|z(t)\|_\Delta^{2\sigma-\tau}\le V_n(z(t))\le \lambda_2\|z(t)\|_\Delta^{2\sigma-\tau} \tag{32}$$

and $$\varpi_1\|z(t)\|_\Delta^{2\sigma}\le U(z(t))\le \varpi_2\|z(t)\|_\Delta^{2\sigma} \tag{33}$$

Moreover, by Lemma 4, we have

$$\begin{aligned}\lambda_2\frac{L}{L}\|z(t)\|_\Delta^{2\sigma-\tau}&=L\left(\left(\frac{\lambda_2}{L}\right)^{1/\tau}\right)^{\tau}\|z(t)\|_\Delta^{2\sigma-\tau}\\ &\le\frac{2\delta-\tau}{2\sigma}L\|z(t)\|_\Delta^{2\sigma}+\frac{\tau L^{(\tau-2\sigma)/\tau}}{2\sigma}\lambda_2^{(\tau-2\sigma)/\tau}\end{aligned} \tag{34}$$

Then, we have

$$V(z(t))\le\rho_2L\|z(t)\|_\Delta^{2\sigma}+\frac{\tau}{2\sigma L^{(2\sigma-\tau)/\tau}}\lambda_2^{\frac{\tau-2\sigma}{\tau}} \tag{35}$$

or

$$\frac{1}{\rho_2}V(z(t))\le L\|z(t)\|_\Delta^{2\sigma}+\frac{\tau}{2\sigma\rho_2L^{(2\sigma-\tau)/\tau}}\lambda_2^{\frac{\tau-2\sigma}{\tau}} \tag{36}$$

where $\rho_2=:\left(\varpi_2+\dfrac{2\delta-\tau}{2\sigma}\right)$.

Therefore, it follows from (29) and (35) that

$$\begin{aligned}\dot V(z(t))&\le-\left(L\|z(t)\|_\Delta^{2\sigma}+\frac{\tau}{2\sigma\rho_2L^{(2\sigma-\tau)/\tau}}\lambda_2^{\frac{\tau-2\sigma}{\tau}}\right)\\ &+\frac{\tau}{2\sigma\rho_2L^{(2\sigma-\tau)/\tau}}\lambda_2^{(\tau-2\sigma)/\tau}+\frac{\rho_1}{L^{1+\gamma}}\\ &\le-\frac{1}{\rho_2}V(z(t))+\bar\rho_1,\end{aligned} \tag{37}$$

where $\bar\rho_1=\dfrac{\tau}{2\sigma\rho_2L^{(2\sigma-\tau)/\tau}}\lambda_2^{(\tau-2\sigma)/\tau}+\dfrac{\rho_1}{L^{1+\gamma}}$.

That is

$$\frac{d}{dt}\left(e^{t/\rho_2}V(z(t))\right)\le e^{t/\rho_2}\bar\rho_1 \tag{38}$$

taking integral on both sides,

$$e^{t/\rho_2}V(z(t))-V(z(0))\le\bar\rho_1\left(e^{t/\rho_2}-1\right) \tag{39}$$

Hence, there exists a $T>0$, for all $t>T$

$$V(z(t))\le e^{-t/\rho_2}V(z(0))+\bar\rho_1\left(1-e^{-t/\rho_2}\right)\le 3\bar\rho_1 \tag{40}$$

from which it is clear that $z_1=y-y_r$ can be rendered smaller than any positive tolerance with a sufficiently large L.

3.3. Extension of main result

In this subsection, using homogeneous domination approach, we will show that the triangular growth condition as required in Assumption 2 is not necessary to achieve the global practical tracking of (1) under the following more general assumption.

Assumption 3.

There are constants $C_1>0, C_2>0, L>1, 0<\gamma_i\le 1$, and $\tau\ge 0$ such that

$$\left|\frac{\varphi_i(\cdot)}{L^{\kappa_i}}\right|\le C_1L^{1-\gamma_i}\sum_{j=1}^{n}\left(\left|\frac{x_j(t)}{L^{\kappa_j}}\right|^{\frac{r_i+\tau}{r_j}}+\left|\frac{x_j\left(t-d_j(t)\right)}{L^{\kappa_j}}\right|^{\frac{r_i+\tau}{r_j}}\right)+\frac{C_2}{L^{\kappa_i}}, \tag{41}$$

where $\kappa_1=0, r_1=1,\ \kappa_{i+1}=(\kappa_i+1)/p_i$, and $r_{i+1}p_i=r_i+\tau>0,\quad i=1,\dots,n$.

Since it is apparent that Assumption 1 implies

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Assumtion 3 from inequality (41), the condition of Assumption 3 is more weaker than that of Assumption 1. Therefore, the following theorem is a more general result achieved under Assumption 3.

Theorem 3. Under Assumptions 2 and 3, the global practical output tracking problem of system (1) can be solved by a state feedback controller of the form (19).

Proof. Similar to (23), Assumption 3 will directly lead to (25). Rest of the proof is similar to that of Theorem 2 and hence is omitted here.

4. AN ILLUSTRATIVE EXAMPLE AND SIMULATION

In this section, we give a simple numerical example to illustrate the correctness and effectiveness of the theoretical results by considering the following nonlinear system

$$\begin{aligned} \dot{x}_1(t) &= x_2^{5/3}(t) + x_1^{1/3}(t-d) \\ \dot{x}_2(t) &= x_3^{5/3}(t) + 2x_2(t) \\ \dot{x}_3(t) &= u(t) + x_3^{1/3}(t) \\ y(t) &= x_1(t) \end{aligned} \tag{42}$$

where $p_1 = 5/3$, $p_2 = 5/3$, $p_3 = 1$ and d represent a time-delay parameter. Our objective is to design a state feedback practical output tracking controller such that the output of the system (42) tracks a desired reference signal y_r, and all the states of the system (42) are globally bounded.

Clearly, the system is of the form (1). It is worth pointing out that although system (42) is simple, it cannot be solved the global practical tracking problem using the design method presented in [4-5] and [7] because of the presence of time-delay term $x_1^{1/3}(t-d)$. Choose $\tau = 2/3$ and $r_1 = 1$, then $r_2 = r_3 = 1$ and $r_4 = 5/3$. Further, choose the reference signal $y_r = \sin(t/3) + \sin t$. Then, by Lemma 4, it is easy to obtain

$$\begin{aligned} |\varphi_1(\cdot)| &= |z_1(t-d)|^{\frac{1}{3}} \le 2^{\frac{4}{3}} |z_1(t-d)|^{\frac{1}{3}} \\ &\le \frac{1}{5}|z_1(t-d)|^{\frac{5}{3}} + \frac{4}{5}2^{\frac{5}{3}} \end{aligned},$$

$$|\varphi_2(\cdot)| = |2z_2| \le \left(2^{3/2}\right)^{2/3}|z_2| \le \frac{3}{5}|z_2|^{5/3} + \frac{2}{5}2^{5/2},$$

$$|\varphi_3(\cdot)| = |z_3|^{1/3} \le 2^{4/3}|z_3|^{1/3} \le \frac{1}{5}|z_3|^{5/3} + \frac{4}{5}2^{5/3}$$

and

$$\begin{aligned} |y_r| &= |\sin(t/3) + \sin t| \le 2, \\ |\dot{y}_r| &= \left|\frac{1}{3}\cos(t/3) + \cos t\right| \le \frac{4}{3} \cdot \end{aligned}$$

Clearly, Assumptions 1-2 holds with $C_1 \ge 3/5$, $C_2 \ge 16/5$ and $D \ge 4$. According to the design procedure proposed in Section 3 (by Theorem2), we can obtain a state feedback tracking controller

$$u = -2L^{49/25}\left(L^{24/25}x_3 + 2\left(L^{3/5}x_2 + 2\left(x_1 - y_r\right)\right)\right)^{5/3} \tag{43}$$

In the simulation, by choosing the initial values as $z_1(\theta) = 3,\ z_2(\theta) = -5,\ z_3(\theta) = -2,\ \theta \in [0, d]$, where $d = 1$ and the reference signal $y_r = \sin(t/3) + \sin t$. Then, we have the following (i) and (ii).

(i) When the scaling gain L is chosen as $L = 100$, the tracking error obtained is about 0.45 as shown in Fig. 1.

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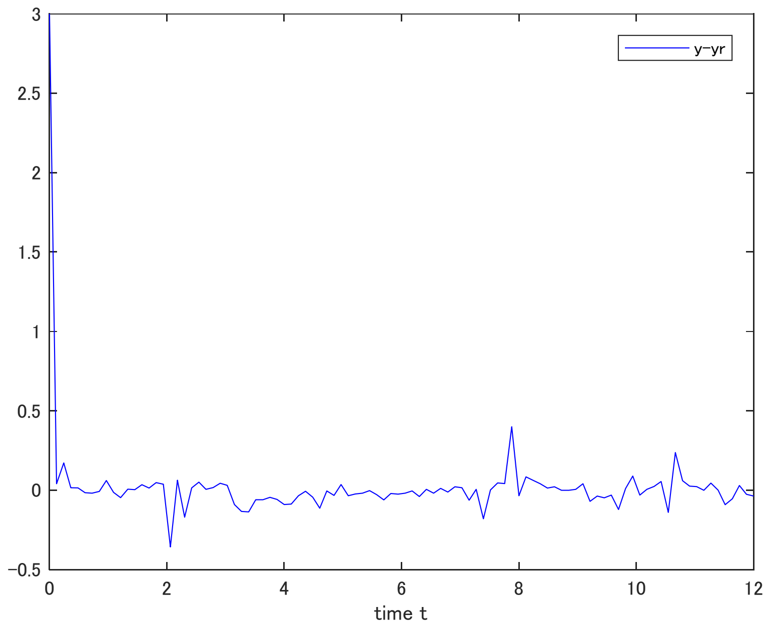

Fig. 1(a). Tracking error $y - y_r$ *for* $L = 100$.

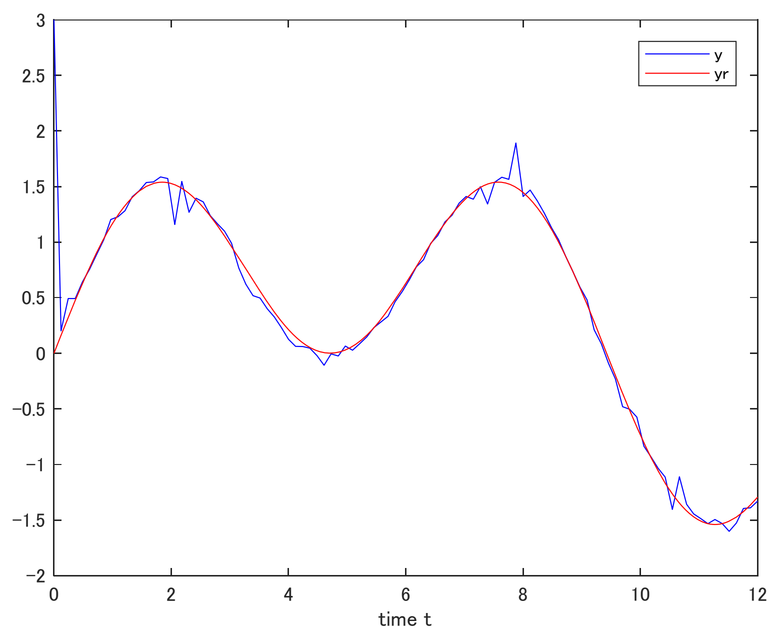

Fig. 1(b). The trajectories of $x_1(t)$, $y_r(t)$ *for* $L = 100$

(ii) When the scaling gain L is chosen as $L = 700$, then the tracking error reduces to about 0.1 as

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shown in Fig. 2.

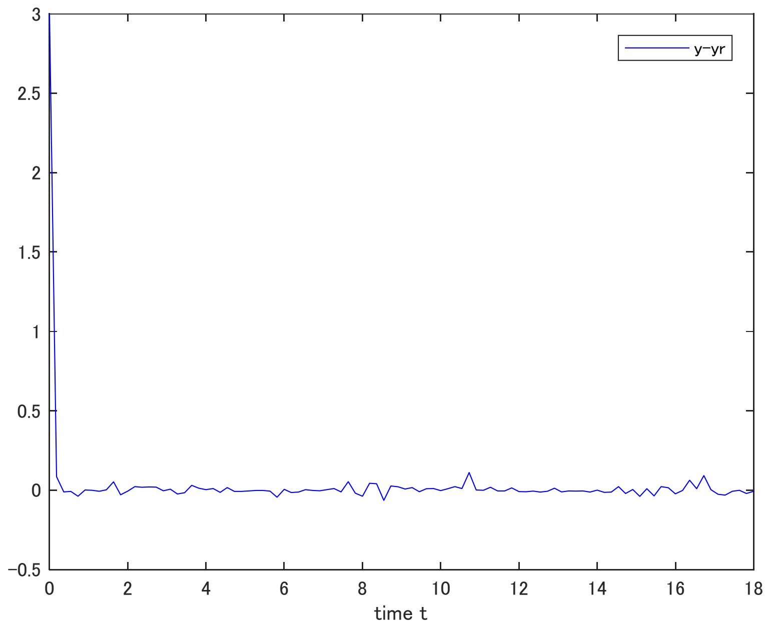

Fig. 2(a). Tracking error $y - y_r$ *for* $L = 700$.

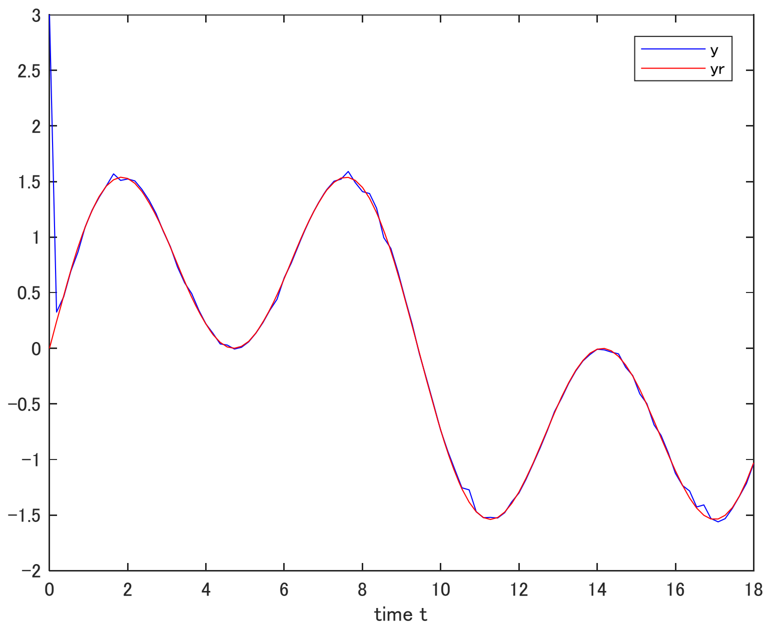

Fig. 2(b). The trajectories of $x_1(t), y_r(t)$ *for* $L = 700$

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5. CONCLUSION

In this paper, we have studied the practical output tracking problem for a class of uncertain high-order time-delay nonlinear systems under a homogeneous condition. First, we design a homogeneous state feedback controllers have been constructed with adjustable scaling gains. Then, with the help of a homogeneous Lyapunov-Krasovskii functional, we've redesigned the homogeneous domination approach to tune the scaling gain for the overall the closed loop systems. It is shown that an appropriate choice of gain will enable us to globally track for a class of uncertain non-linear systems in finite time. Moreover, the proposed approach can also widen the applicability to a broader class of systems with non-triangular structure. It should be noted that the proposed controller can only work well when the whole state vector is measurable. Therefore, a natural and more interesting problem is how to design feedback output tracking controller for the time-delay nonlinear systems studied in the paper if only partial state vector being measurable, which is now under our further investigation.

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